Volume Stabilization and Acceleration in Brane Gas Cosmology

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Abstract

We investigate toy cosmological models in $(1+m+p)$-dimensions with gas of p-branes wrapping over p-compact dimensions. In addition to winding modes, we consider the effects of momentum modes corresponding to small vibrations of branes and find that the extra dimensions are dynamically stabilized while the others expand. Adding matter, the compact volume may grow slowly depending on the equation of state. We also obtain solutions with winding and momentum modes where the observed space undergoes accelerated expansion.

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I. INTRODUCTION

String/M theory predicts six/seven extra dimensions which are presumably compact and very small compared to the observed three dimensions. To find out how extra dimensions are compactified is a major problem in the theory waiting for a solution. This is important in determining the vacuum structure and the low energy content which may then be compared with experimental findings to test validity of string/M theory. It is known that phenomenologically viable models can be obtained in Calabi-Yau or G2 manifold compactifications.

Cosmology, on the other hand, is another framework where ideas in string/M theory may find observational support. We know that the perceived universe was initially small and grew to its size today after various cosmological eras. If extra dimensions exist they should have an effect on the cosmological evolution. However, since standard cosmology offers a successful scenario which remains plausible close to the temperatures around Tev scale, the effects of extra dimensions should be negligible from that time till today. For example, the theory of big-bang nucleosynthesis together with the observational abundances of light elements provide a limit to the change in the size of internal space following the production of these elements. Therefore, it is natural to assume that the extra dimensions were already stabilized around the epoch of nucleosynthesis.

A mechanism for stabilization based on the cosmological impact of string winding and momentum modes was proposed in [1] (see also [2] and [3] for earlier work on string winding modes in cosmology). In the same paper, it is suggested that the initial big-bang singularity can be resolved by T-duality invariance of string theory and the dimensionality of observed space-time can be explained via the annihilation of winding strings leading decompactification of three dimensions. The proposal of [1] was generalized in [4] to include other branes in string/M theory and it is now an alternative approach named as brane gas cosmology (for recent work see e.g. [5–22]). In this scenario, strings dominate the the late time evolution and thus the stabilization of extra dimensions *today* should be achieved by string winding and momentum modes. This is quantified in the papers [19–22] where solutions to Einstein and dilaton gravity equations with stabilized internal dimensions are presented.

However, this mechanism seems to work for toroidal compactifications which are not phenomenologically viable. For example, it is not clear how strings may stabilize *volume* of a Calabi-Yau or a G2 manifold since the topology is not equal to the product of one-

dimensional cycles (this is much more evident for a sphere compactification since there cannot be stable winding strings at all). Therefore, to fix the moduli related to higher dimensional non-trivial (i.e. non-toroidal) topological cycles, it seems that the higher dimensional branes are also needed to play a cosmological role at late times.

Motivated by this argument, in this paper we consider a *toy* cosmological model in $(1+m+p)$-dimensions with a gas of p-branes wrapping over the p-dimensional compact space, and try to determine whether the compact volume is stabilized. We should emphasize that such a model does not straightforwardly lead to a realistic scenario. Apart from the fact that all other possible (stringy) excitations are ignored, the existence of a gas of 6 or 7-branes is also dubious (note that in a realistic model one should set $m = 3$ and $p = 6, 7$) since in the very early universe the higher dimensional branes are expected to annihilate leaving a gas of lower dimensional branes at late times. In any case, the results obtained in such toy models are still expected to be useful in brane gas cosmology.

As in the original proposal of [1], one would anticipate a balance between *brane* winding and momentum pressures for stabilization. The energy momentum tensor for winding part was already determined in the cosmological context (see e.g. [9] or [14]). Since we do not know how to quantize p-branes for $p > 1$, the exact spectrum of the momentum modes is not known. However, in a semiclassical approximation one may consider small fluctuations around a rigid wrapped p-brane and from this spectrum an energy momentum tensor for momentum modes can be determined. In section II, we will work out this problem and study the issue of stability in the context of Einstein gravity. We find that, like strings, gas of p-branes with $p > 1$ can also dynamically stabilize extra dimensions.

Based on recent observations one may claim that any potentially realistic cosmological model should include a theory for accelerated expansion. It is well known that inflationary paradigm is successful in resolving the basic shortcomings of standard cosmology. On the other hand, there are strong observational indications that the universe undergoes an accelerated expansion today. In brane gas cosmology, a way of realizing power-law inflation driven by a gas of domain walls was proposed in [23]. In section III, we also obtain accelerating solutions with brane winding and momentum modes (the background of winding modes generalizes the power-law metric constructed in [13]). As we will discuss, depending on the initial conditions, one encounters two different behavior: either a short period of acceleration with a small number (like 0.7) of e-foldings or an infinite accelerated expansion

ending at a singularity at finite proper time.

II. STABILIZATION

Consider a $(1+m+p)$-dimensional space-time with the metric

$$ds^2 = -dt^2 + dx^i dx^i + R^2\, d\Sigma_p^2, \tag{1}$$

where $i, j = 1, .., m$, Σ_p is a p-dimensional compact manifold, R is a constant scale factor and the metric on Σ_p can be written as $(a, b = 1, .., p)$,

$$d\Sigma_p^2 \;=\; g_{ab}(y)\, dy^a dy^b. \tag{2}$$

In this paper we take g_{ab} to be Ricci flat. For other cases, we expect the internal curvature terms to become important at late times as in [15]. In the physical gauge (and for small curvatures) the action for a p-brane wrapping over Σ_p can be written as

$$\begin{aligned} S_p \;&=\; -T_p \int d^{p+1}\sigma\, \sqrt{-\gamma}, \\ &=\; -T_p \int d^{p+1}\sigma\, \sqrt{-\gamma_0} \;-\; \frac{T_p}{2} \int d^{p+1}\sigma\, \sqrt{-\gamma_0}\, \gamma_0^{\alpha\beta}\, \partial_\alpha x^i\, \partial_\beta x^i \;+\; \mathcal{O}(x^4), \end{aligned} \tag{3}$$

where T_p is the brane tension, $\sigma^\alpha = (t, y^a)$ are the local brane coordinates, $\gamma^0_{\alpha\beta}$ is the zeroth order induced metric given by

$$\gamma^0_{\alpha\beta}\, d\sigma^\alpha\, d\sigma^\beta \;=\; -dt^2 \;+\; R^2\, d\Sigma_p^2, \tag{4}$$

and in the last line in (3) we expand the action to the second order in the transverse fluctuation fields $x^i(\sigma^\alpha)$.

The first term in (3) corresponds to the winding modes which have the energy spectrum

$$E \;=\; (n\, T_p\, \Omega_p)\, R^p, \tag{5}$$

where n is winding number and Ω_p is the volume of Σ_p. From (5), the total pressure in the internal space can be found as

$$P \;=\; -p\, (n\, T_p\, \Omega_p)\, R^p, \tag{6}$$

which turns out to be negative, as expected.

On the other hand, (3) yields the following field equation for x_i:

$$\ddot{x}_i - \frac{1}{R^2}\,\nabla^2\, x_i = 0, \tag{7}$$

where the dot denotes derivative with respect to t and ∇^2 is the Laplacian on Σ_p. Expanding in terms of the harmonics on Σ_p, the spectrum of *momentum* modes (i.e. the eigenfrequencies $e^{i\omega t}$) can be obtained as $\omega^2 = \lambda_n^2/R^2$, where $-\lambda_n^2$ are the eigenvalues of the Laplacian. Thus the energy for the n'th momentum mode is given by (we choose $\lambda_n > 0$)

$$E \;=\; \frac{\lambda_n}{R}, \tag{8}$$

and the corresponding total pressure on Σ_p is

$$P \;=\; \frac{\lambda_n}{R}, \tag{9}$$

which is positive contrary to (6). Although (8) and (9) depend on the undetermined eigenvalues λ_n, one does not need these numbers to proceed. On the other hand, $1/R$ dependence in (9) can also be deduced from the uncertainty principle $\Delta y\,\Delta P \sim 1$ where a compact direction y in Σ_p approximately satisfies $\Delta y \sim R$. This suggests that (8) and (9) should also hold for other possible fields propagating on the world-volume such as U(1) gauge fields on D-branes or self-dual 3-form of M5 brane. Gauging away the longitudinal polarizations, the linearized transverse modes of these brane fields are expected to obey (7).

In a cosmological setting, (1) should be replaced with

$$ds^2 = -e^{2A}dt^2 + e^{2B}dx^i dx^i + e^{2C}\, d\Sigma_p^2, \tag{10}$$

where the metric functions A, B and C depend only on time t. The scale factors for the observed and the internal spaces are defined as

$$R_{ob} = e^B, \qquad R_{in} = e^C. \tag{11}$$

Note that t is *not* the proper time and we have not yet fixed the corresponding reparametrization invariance.

From (5)-(9), the energy momentum tensor for the brane winding and momentum modes can be obtained by calculating the corresponding *densities* (i.e. by dividing with the total spatial volume $V = e^{mB+pC}$) which gives

$$T_{\hat{t}\hat{t}} \;=\; T_w\, e^{-mB} \;+\; T_m\, e^{-mB-(p+1)C},$$

$$
\begin{aligned}
T_{\hat{i}\hat{j}} \; &= \; 0, \\
T_{\hat{a}\hat{b}} \; &= \; -T_w \, e^{-mB} \, \delta_{ab} \, + \, \frac{T_m}{p} \, e^{-mB-(p+1)C} \, \delta_{ab},
\end{aligned}
\tag{12}
$$

where T_w and T_m are constants (which can in principle be fixed by the parameters of the underlying fundamental theory and thermodynamics) and the hatted indices refer to the tangent space. It is easy to verify that $\nabla_\mu T^{\mu\nu} = 0$. In driving (12), the total pressures given in (6) and (9) are assumed to be distributed isotropically on the tangent space of Σ_p since the momentum modes are point-like excitations (note that we are doing field theory on the world-volume) and thus locally they would only see the flat, tangent space. However, to justify this assumption more rigorously one may additionally require that Σ_p is not highly curved and thus has a "smooth" shape with no sharp corners or handles, which is also necessary for the validity of the brane action and the small field approximation used in (3). Let us note that the contribution of the winding part in (12) agrees with the expressions derived in [13, 14] which are obtained in a different way by coupling the brane action to the gravity action. This agreement further supports the above considerations.

In general, if one has the metric

$$
ds^2 = -e^{2A} dt^2 + \sum_i e^{2B_i} dx^i dx^i, \tag{13}
$$

and the energy momentum tensor

$$
T_{\hat{\mu}\hat{\nu}} = \mathrm{diag}\,(\rho, p_{\hat{i}}), \tag{14}
$$

where $p_{\hat{i}} = \omega_i \, \rho$ with constant ω_i, the conservation equation $\nabla_\mu T^{\mu\nu} = 0$ gives

$$
\rho = \rho_0 \; \exp\left[-\sum_i (1+\omega_i) B_i \right] , \tag{15}
$$

where ρ_0 is a constant. Comparing to (12), we find that the energy momentum tensor for winding and momentum modes can be written as (14) where for winding modes

$$
\omega_i = \begin{cases} -1 : & \text{brane direction,} \\ \;\;0 \;\; : & \text{transverse direction,} \end{cases} \tag{16}
$$

and for momentum modes

$$
\omega_i = \begin{cases} 1/p : & \text{brane direction,} \\ \;\;0 \;\; : & \text{transverse direction,} \end{cases} \tag{17}
$$

which corresponds to radiation confined in the compact space.[1] With the above formulas it is possible to determine the energy momentum tensor for any given brane configuration and study the resulting cosmological evolution.

Using (12) and imposing the gauge $A = m\,B + p\,C$ in (10), which fixes the t-reparametrization invariance, Einstein equations can be written as follows:

$$\begin{aligned} &\ddot{A} - \dot{A}^2 + m\,\dot{B}^2 + p\,\dot{C}^2 = -\frac{m-2}{m+p-1}\,T_w\,e^{mB+2pC} - T_m\,e^{mB+(p-1)C}, \\ &\ddot{B} = \frac{p+1}{m+p-1}\,T_w\,e^{mB+2pC}, \\ &\ddot{C} = -\frac{m-2}{m+p-1}\,T_w\,e^{mB+2pC} + \frac{T_m}{p}\,e^{mB+(p-1)C}, \end{aligned} \tag{18}$$

where the dot denotes derivative with respect to t and the gravitational constant is set to one.

We could not obtain the most general solution of (18). However, it is easy to verify that the functions

$$A = -2\ln(\alpha\,t) + p\,C_0, \qquad B = \frac{-2}{m}\ln(\alpha\,t), \qquad C = C_0, \tag{19}$$

give a solution where α is a constant, which is uniquely[2] fixed by field equations, and C_0 can be calculated from the third equation as

$$e^{C_0} = R_{in}^0 = \left[\frac{T_m\,(m+p-1)}{T_w\,(m-2)\,p}\right]^{1/(p+1)}. \tag{20}$$

Introducing the proper time coordinate

$$d\tau = -e^A\,dt, \tag{21}$$

we get

$$ds^2 \;=\; -d\tau^2 \;+\; (\alpha\,\tau)^{4/m}\,dx^i dx^i \;+\; (R_{in}^0)^2\,d\Sigma_p^2, \tag{22}$$

where the internal dimensions are stabilized and the power-law of the observed space is exactly the same as the one for pressureless matter in standard cosmology. Here, R_{in}^0 can

[1] The energy-momentum tensor for (16) or (17) can also be viewed to correspond to a perfect fluid in $(1+m+p)$-dimensions. Certain aspects of higher dimensional cosmological models with general multi-component perfect fluid matter were studied in the past, see e.g. [24–27].

[2] The field equations give

$$\alpha^2 = \frac{m(p+1)T_m}{2p(m-2)}\,(R_{in}^0)^{p-1}.$$

be viewed as the self-dual radius corresponding to brane winding and momentum modes. Technically, C is allowed to be a constant since in the third equation in (18), $\exp(mB)$ terms can be factored out and there are both negative and positive contributions to $\ddot{C}$.

The metric (22) corresponds to the initial data where the internal dimensions begin at the self-dual radius with zero velocity. For more general initial conditions one may integrate (18) numerically. The first equation (when combined with the other two) in (18) gives a constraint on the initial data:

$$m(m-1)\left(\frac{dB}{d\tau}\right)^2+p(p-1)\left(\frac{dC}{d\tau}\right)^2+2mp\left(\frac{dB}{d\tau}\right)\left(\frac{dC}{d\tau}\right)-2T_we^{-mB}-2T_me^{-mB-(p+1)C}=0, \tag{23}$$

where τ is the proper time defined in (21). Fixing $B(\tau)$, $C(\tau)$ and one of the velocities[3] $dB/d\tau$ or $dC/d\tau$ at some $\tau=\tau_0$, (23) gives *two real* roots for the undetermined initial velocity. It is easy to see that one of the roots should have the opposite sign compared to the other fixed initial velocity.

After several numerical integrations of (18) for different initial conditions and for $(m,p)=(3,6)$ (these (m,p) values are suggested by string theory, however we expect the same conclusions to hold for any set with $m>2$) we observe the following generic behavior:

(i) For one of the roots, $\exp(C)=R_{in}$ performs damped oscillations around the self-dual radius given in (20) and the observed space expands with negative acceleration close to the power-law in (22). As the extra dimensions are stabilized asymptotically, the solution becomes (22). In figure 1, we plot two such numerical runs for R_{in}. As in [19], the evolution of the large dimensions is responsible for these oscillations to be damped.

(ii) For the other root, the numerical integration signals a singularity at finite proper time, where the observed and the internal spaces have the opposite behavior near the singularity i.e. if one expands the other contracts. Below, we will try to explain this case analytically.

(iii) If the initial values of $dB/d\tau$ and $dC/d\tau$ are positive (i.e. if both spaces are expanding at the beginning) then we observe the stabilization mentioned in (i). For the other cases both (i) and (ii) are possible.

It is not surprising that for some initial conditions the metric ends with a singularity at finite proper time. We do not expect to obtain a completely smooth cosmological solution with the ansatz (10). Contrary, at least one singular fixed point in the past or in the future

[3] In our numerical integrations, we first fix $dC/d\tau$ and solve $dB/d\tau$ from (23).

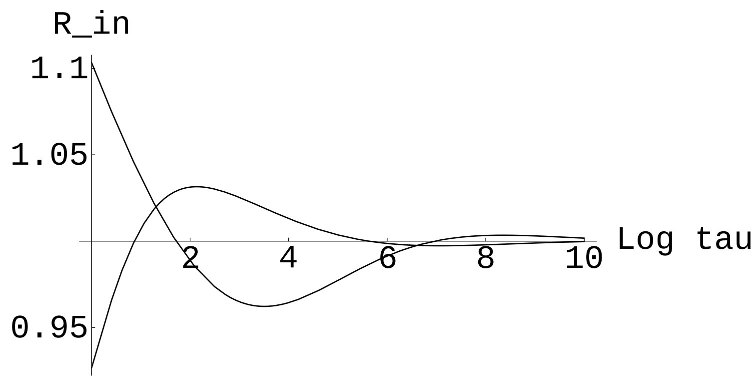

FIG. 1: The graphs of $R_{in}(\tau)$ for two different initial data set obeying (i). For convenience we take $m = 3$, $p = 6$, $T_w = 8$ and $T_m = 6$ so that the self-dual radius (20) is equal to 1. The initial conditions are: $R_{ob}(0) = 1$, $R_{in}(0) = 0.7$, $dR_{ob}/d\tau(0) = 3.9761$, $dR_{in}/d\tau(0) = 0.3$ and $R_{ob}(0) = 1$, $R_{in}(0) = 1.3$, $dR_{ob}/d\tau(0) = 3.65135$, $dR_{in}/d\tau(0) = -0.7$. Here, $dR_{ob}/d\tau(0)$ is solved from (23) given other conditions.

is anticipated. Therefore, for the alternatives (i) and (ii) the following interpretation can be given: in the phase space one of the roots selects a trajectory which evolves "forwards in time" and gives the smooth stabilized solution, and the other root corresponds to a path where the space-time evolves "backwards in time" and hits the initial singularity.

Actually, to define the arrow of time in such solutions an extra (observational) input is needed. For instance, we use the Robertson-Walker metric starting from the big-bang singularity since we observe that the universe at the moment is expanding. If we would measure a contraction rather than an expansion, the same metric should be considered with $\tau \to -\tau$, i.e. with a future big-crunch singularity instead. In our case, there is no such avaliable observational input. However, it seems reasonable to assume that (18) holds some time after the big-bang explosion and thus one should take all dimensions to be expanding initially in (18). From (iii) we then see that the corresponding solution asymptotes to (22) and the stabilization can be achieved. On the other hand, we should also bear in mind that there are physically acceptable initial data, such as an expanding observed and contracting internal dimensions at the beginning, where the subsequent evolution ends with a naked singularity at finite proper time.

To have a better understanding of this singular behavior, let us analyze (18) more care-

fully. Comparing the contributions of the winding and momentum modes, we see that when $\exp[(p{+}1)C] \gg T_m/T_w$ the winding modes and when $\exp[(p{+}1)C] \ll T_m/T_w$ the momentum modes dominate the evolution. So, it would be useful to solve (18) in these two limits.

To consider the effects of winding modes alone we set $T_m = 0$. Then the last two equations in (18) implies

$$C \;=\; -\frac{m-2}{p+1}B \;+ a\,t, \tag{24}$$

where a is a constant. Using this relation in (18), one gets a second order differential equation for B which can be solved (up to an irrelevant additive constant depending on T_w) as

$$B = -\frac{2(p+1)}{m-mp+4p}\,\ln(\sinh(b\,t)) - \frac{2p(p+1)}{m-mp+4p}\,a\,t \tag{25}$$

where b is another constant. Using (24), (25) and the gauge $A = m\,B + p\,C$ in the first equation in (18) one finds

$$a^2 = \frac{4(m+p-1)}{mp(p+1)^2}\,b^2, \tag{26}$$

and this completes the solution. In terms of the proper time coordinate (21), we see that as $t \to 0$, $\tau \to \infty$ and as $t \to \infty$, $\tau \to 0$. In these two limits the solution becomes

$$ds_w^2 = \begin{cases} -d\tau^2 + (\tau)^{2\,r_1}\,dx^i\,dx^i \;+\; (\tau)^{2\,r_2}\,d\Sigma_p^2 & :\text{as}\quad \tau \to 0, \\ \\ -d\tau^2 + (\tau)^{\frac{4}{m}}\,dx^i\,dx^i \;+\; (\tau)^{-\frac{4(m-2)}{m(p+1)}}\,d\Sigma_p^2 & :\text{as}\quad \tau \to \infty, \end{cases} \tag{27}$$

where the powers r_1 and r_2 are given by

$$r_1 \;=\; \frac{m \;+\; \epsilon\sqrt{m\,p\,(m+p-1)}}{m(m+p)}, \tag{28}$$

$$r_2 \;=\; \frac{p \;-\; \epsilon\sqrt{m\,p\,(m+p-1)}}{p(m+p)}. \tag{29}$$

Here, $\epsilon = \pm 1$ corresponds to the positive and the negative roots of (26), respectively, and in (27) we ignore constants multiplying τ in parenthesis which can be set to 1 by scalings of x and y coordinates. We see that the full metric smoothly interpolates between these two limiting geometries.

Asymptotically, as $\tau \to \infty$ the solution approaches the power-law metric given in [13] (see eq. (11) in that paper). On the other hand, as $\tau \to 0$ one finds a vacuum Kasner metric since the powers r_1 and r_2 obey

$$m\,r_1 \;+\; p\,r_2 = 1, \tag{30}$$

$$m\,r_1^2 \;+ p\,r_2^2 = 1. \tag{31}$$

Indeed, using (24) and (25) it is easy to see that *all* the source terms on the right hand side of (18) vanish as $t \to \infty$ (and thus $\tau \to 0$) so ending up with a vacuum solution is not surprising. This also shows that the same vacuum metric solves (18) with $T_m \neq 0$ in the $\tau \to 0$ limit.

To see the effects of momentum modes alone let us now take $T_w = 0$. In this case the second equation in (18) gives

$$B = a\,t, \tag{32}$$

where a is a constant, and the third equation can be solved for C (up to an irrelevant additive constant depending on T_m) as

$$C = -\frac{2}{p-1}\,\ln(\sinh(b\,t)) - \frac{m}{p-1}\,a\,t. \tag{33}$$

The unknown function A can be fixed from the gauge condition $A = m\,B + p\,C$, and the first equation in (18) imposes

$$a^2 = \frac{4p}{m(m+p-1)}\,b^2. \tag{34}$$

In terms of the proper time (21), we find that as $t \to (0,\infty)$, $\tau \to (\infty, 0)$, respectively, and asymptotically the solution becomes

$$ds_m^2 = \begin{cases} -d\tau^2 + (\tau)^{2\,r_1}\,dx^i\,dx^i + (\tau)^{2\,r_2}\,d\Sigma_p^2 & : \text{as} \quad \tau \to 0, \\ \\ -d\tau^2 + dx^i\,dx^i + (\tau)^{\frac{4}{p+1}}\,d\Sigma_p^2 & : \text{as} \quad \tau \to \infty, \end{cases} \tag{35}$$

where r_1 and r_2 are given by (28) and (29) in which $\epsilon = +1$ corresponds to the negative root of (34) while $\epsilon = -1$ corresponds to the positive root. As $\tau \to 0$, one again encounters the *same* vacuum Kasner metric.

Returning back the situation where both T_w and T_m are non-zero, we see from (27) and (35) that as $\tau \to 0$ the solution should approach the Kasner background with the powers r_1 and r_2. On the other hand, the field equations show that as $\tau \to \infty$ the metrics given in (27) and (35) should be modified i.e. in (27) momentum modes can no longer be ignored and in (35) winding modes should be taken into account. In either case the neglected contribution becomes dominant in time, alters the behavior of the extra dimensions and serves for stabilization. This suggests that asymptotically the most general solution of (18)

should become

$$ds^2 = \begin{cases} -d\tau^2 + (\tau)^{2\,r_1}\, dx^i\, dx^i \;+\; (\tau)^{2\,r_2}\, d\Sigma_p^2 & : \text{as} \quad \tau \to 0, \\ \\ -d\tau^2 \;+\; (\tau)^{\frac{4}{m}}\, dx^i\, dx^i \;+\; (R_{in}^0)^2\, d\Sigma_p^2 & : \text{as} \quad \tau \to \infty. \end{cases} \tag{36}$$

The numerical integrations of (18), which are summarized in (i)-(iii) above, also support this result. We already mentioned in (i) that for some initial conditions the metric approaches to (22). The singularity corresponding to other initial data indicated in (ii) occurs at $\tau = 0$ in (36). Recall from (ii) that in reaching the singularity the observed and the internal spaces have the opposite behavior. From (28) and (29), we see that r_1 and r_2 have the opposite signs and thus (36) also explains this numerical result. As discussed above, such initial conditions are associated with solution evolving "backwards in time".

Having studied the stabilization problem with brane winding and momentum modes, let us now try to include the effects of ordinary matter. These should be taken into account in determining the late time behavior since in the universe today the existence of radiation and pressureless dust is known. We take the following energy momentum tensor for matter

$$T_{\hat{\mu}\hat{\nu}} = \text{diag}(\, \rho,\, p_{\hat{i}},\, p_{\hat{a}}\,), \tag{37}$$

where

$$p_{\hat{i}} = \omega\, \rho, \qquad\qquad p_{\hat{a}} = \nu\, \rho, \tag{38}$$

and (ω, ν) are constants. Adding (37), the Einstein equations (18) are modified as follows (we again impose the gauge $A = m\, B + p\, C$):

$$\begin{aligned} \ddot{A} - \dot{A}^2 + m\,\dot{B}^2 + p\,\dot{C}^2 &= -\frac{m-2}{m+p-1}\, T_w F_w - T_m F_m \;+\; \rho_0 \left[\frac{2 - m(\omega+1) - p(\nu+1)}{m+p-1}\right] F_\rho \\ \ddot{B} &= \frac{p+1}{m+p-1}\, T_w\, F_w \;+\; \frac{\rho_0}{m+p-1}\, [1 + (p-1)\omega - p\nu]\, F_\rho, \\ \ddot{C} &= -\frac{m-2}{m+p-1}\, T_w\, F_w + \frac{T_m}{p}\, F_m \;+\; \frac{\rho_0}{m+p-1}\, [1 + (m-1)\nu - m\omega]\, F_\rho, \end{aligned} \tag{39}$$

where

$$\begin{aligned} F_w &= e^{mB+2pC}, \\ F_m &= e^{mB+(p-1)C}, \\ F_\rho &= e^{(1-\omega)mB+(1-\nu)pC}. \end{aligned} \tag{40}$$

It is difficult to solve (39) in its most general form. However, for $\omega = 0$, there is again a special solution which can be written in the proper time coordinate as:

$$ds^2 \, = \, -d\tau^2 \, + \, (\alpha \, \tau)^{4/m} \, dx^i dx^i \, + \, (R^0_{in})^2 \, d\Sigma_p^2, \tag{41}$$

where the self-dual radius R^0_{in} obeys

$$\frac{m-2}{m+p-1} \, T_w \, (R^0_{in})^{p+1} - \frac{1+(m-1)\nu}{m+p-1} \, \rho_0 \, (R^0_{in})^{1-\nu p} - \frac{T_m}{p} = 0, \tag{42}$$

and α is a fixed[4], positive constant.

The metric (41) evolves from the initial data where the internal space starts at the self-dual radius with zero velocity. For more general initial conditions, we numerically integrate[5] (39) and our findings can be summarized as follows:

(a) $\omega = 0$: We find that the internal dimensions are stabilized after some damped oscillations around the self-dual radius determined in (42) and the observed space expands. Asymptotically, the solution approaches to (41). In figure 2(a), two such numerical integrations for $R_{in}(\tau)$ are plotted. Note that, from (42) the final size of the extra dimensions depends on ν.

(b) $\omega > 0$: We find that the effect of ordinary matter on the evolution is negligible at late times, i.e. brane winding and momentum modes dominate the cosmology. The solution asymptotically becomes (22) where the self-dual radius given by (20) does not depend on ν. In figure 2(b), we plot two numerical runs for $R_{in}(\tau)$. The initial conditions are chosen so that $R^0_{in} = 1$.

(c) $\omega < 0$: In this case, the late time evolution is dominated by matter and the stabilization *cannot* be achieved. Two numerical runs for $R_{in}(\tau)$ are plotted in figure 2(c). At early times, the oscillations caused by winding and momentum modes can be seen in the figure. However, matter finally causes the internal space to grow at late times.

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[4] The field equations impose

$$\alpha^2 = \frac{m(p+1)T_m}{2p(m-2)} \, (R^0_{in})^{p-1} + \frac{m(\nu+1)\rho_0}{2(m-2)} \, (R^0_{in})^{p-\nu p}.$$

[5] As before, we first fix all but the velocity of the observed space at $\tau = 0$ and solve $dB(\tau)/d\tau$ from the constraint equation which gives two real roots. In the following, we only consider one of the roots which yields an evolution "forwards in time".

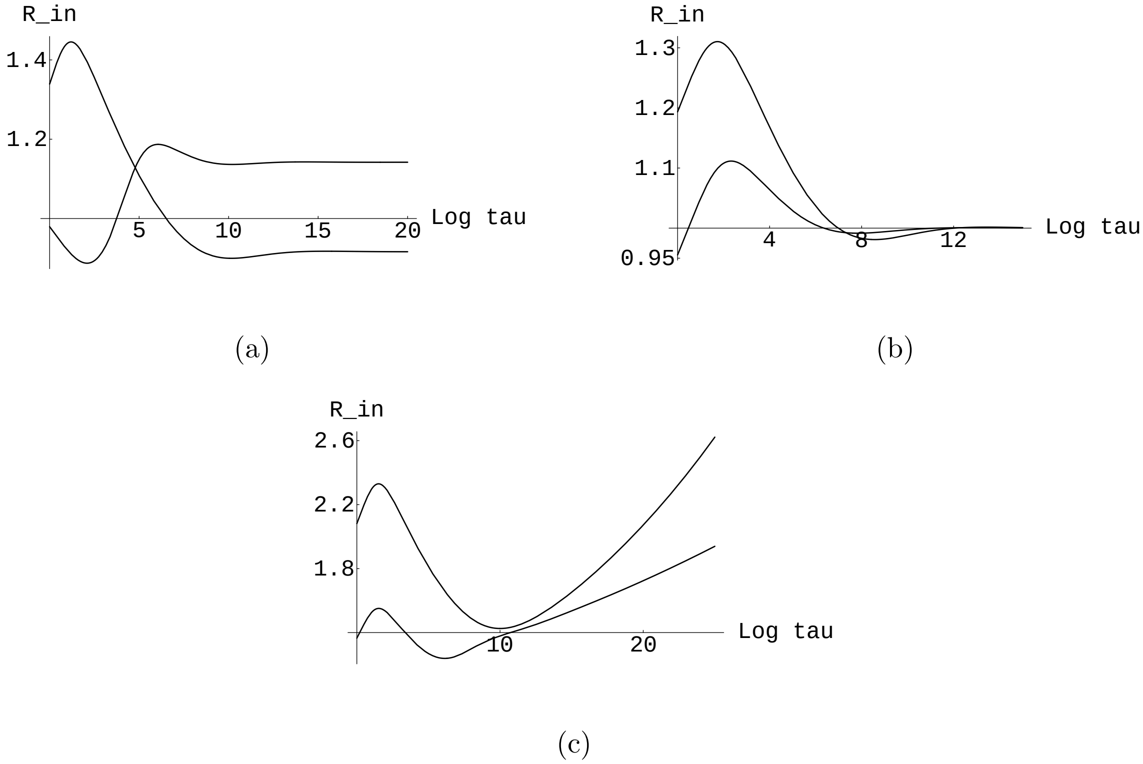

FIG. 2: The graphs of $R_{in}(\tau)$ for different values of ω and ν. For convenience we take $m = 3$, $p = 6$, $T_w = 8$, $T_m = 6$, $R_{ob}(0) = 2$. The initial conditions are:

(a) $[\omega = 0]$: $\nu = -0.9$, $R_{in}(0) = 1.1$, $dR_{ob}/d\tau(0) = -1.41365$, $dR_{in}/d\tau(0) = 1$;

$\nu = 0.5$, $R_{in}(0) = 1.1$, $dR_{ob}/d\tau(0) = 6.80001$, $dR_{in}/d\tau(0) = -0.7$,

(b) $[\omega = 0.5]$: $\nu = -0.6$, $R_{in}(0) = 1.01$, $dR_{ob}/d\tau(0) = 1.10393$, $dR_{in}/d\tau(0) = 0.1$;

$\nu = 0.3$, $R_{in}(0) = 0.8$, $dR_{ob}/d\tau(0) = 1.07248$, $dR_{in}/d\tau(0) = 0.3$,

(c) $[\omega = -0.1]$: $\nu = -0.3$, $R_{in}(0) = 1.7$, $dR_{ob}/d\tau(0) = -0.870317$, $dR_{in}/d\tau(0) = 1$;

$\nu = 0.4$, $R_{in}(0) = 1.1$, $dR_{ob}/d\tau(0) = -1.26295$, $dR_{in}/d\tau(0) = 0.9$.

Here, $dR_{ob}/d\tau(0)$ is solved from the constraint on initial data.

From (39), we see that the contributions of matter, the winding and the momentum modes are proportional to the functions F_ρ, F_w and F_m given in (40), respectively. Assuming that the size of the observed space becomes much larger than the size of the extra dimensions in time[6], one sees that e^B factors in (40) dictate a hierarchy between F-functions. This explains

[6] This is expected since the winding modes resist the expansion of the extra dimensions, however all modes contribute to the growth of the observed space.

the generic behavior we encountered in (a)-(c): compared to F_w and F_m, the function F_ρ gets stronger for $\omega < 0$ and it becomes negligible for $\omega > 0$. In the former case it is clear that the stabilization mechanism does not work. When $\omega = 0$ all functions have the same strength and stabilization can be achieved due to the presence of positive and negative contributions to $\ddot{C}$ in (39), where the self-dual radius now depends on the equation of state. These considerations suggest that if the acceleration of the universe is due to some form of matter with $\omega < 0$ (i.e. dark energy), then the extra dimensions cannot be stabilized by brane winding and momentum modes. We expect the same conclusion to hold for the strings studied in [19]. However, depending on the actual values of the parameters in these equations, the change in the size of the extra dimensions may be slow enough for us not to recognize it in our time scale.

III. ACCELERATION

In this section, we show that some of the solutions constructed above with $T_m = 0$ or $T_w = 0$ have periods during which the observed space undergoes accelerated expansion. Here, we do not intend to propose a realistic model for inflation or current acceleration of the universe, but simply note an alternative way of obtaining acceleration in brane gas cosmology. Motivated by string/M theory as the natural fundamental framework, in this section we also set $m = 3$ and $p = 6$ for convenience.

Let us consider the solution for the brane winding modes given in (24)-(25). Setting $b = 1$ and choosing the negative root[7] in (26), the metric functions become

$$\begin{aligned} A &= -\frac{10}{3}\,\ln(\sinh t) + \frac{8}{3}\,t, \\ B &= -\frac{14}{9}\,\ln(\sinh t) + \frac{16}{9}\,t, \\ C &= \frac{2}{9}\,\ln(\sinh t) - \frac{4}{9}\,t, \end{aligned} \tag{43}$$

where the additive constants, which can be shifted by scaling out x and y coordinates in (10), are set to zero. Eq. (21) can be integrated to fix the proper time:

$$\tau = \frac{3}{14}\,[\cosh(t/3) - \sinh(t/3)]\,[-21 + 23\,\cosh(2t) + 5\,\sinh(2t)]\,\sinh(t)^{-7/3}. \tag{44}$$

[7] For the positive root, we find that there is no period of accelerated expansion.

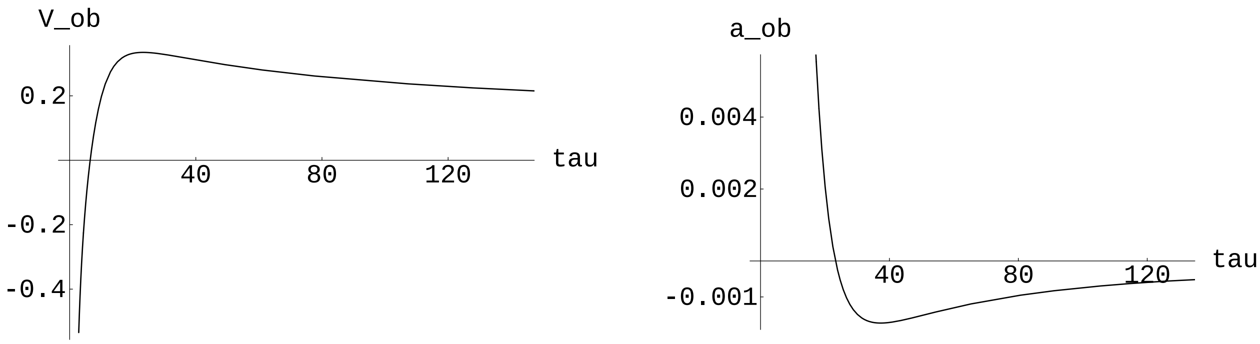

FIG. 3: The graphs of v_{ob} in (45) and a_{ob} in (46) with respect to the proper time τ in (44).

As pointed out before, in the limits $t \to 0, \infty$ we have $\tau \to \infty, 0$, respectively. Using (43), it is easy to calculate the proper velocity and the acceleration of the observed space

$$v_{ob} = \frac{dR_{ob}}{d\tau} = \frac{2}{9} e^{-8t/9} \left[-8 + 7\coth(t)\right] \sinh(t)^{16/9}, \tag{45}$$

$$a_{ob} = \frac{d^2 R_{ob}}{d\tau^2} = -\frac{2}{81} e^{-32t/9} \left[-39 + 88\cosh(2t) - 92\sinh(2t)\right] \sinh(t)^{28/9}, \tag{46}$$

where the scale factors R_{ob} and R_{in} are defined in (11). In figure 3, we plot v_{ob} and a_{ob} versus proper time τ.

From (44), (45) and (46) one finds that $v_{ob} = 0$ at $\tau = \tau_1 \sim 6.5$ and $a_{ob} = 0$ at $\tau = \tau_2 \sim 23.3$. Moreover, v_{ob} is positive for $\tau > \tau_1$ and a_{ob} is positive for $\tau < \tau_2$ (see figure 3). Therefore, in the interval $\tau_1 < \tau < \tau_2$ we have both $v_{ob} > 0$ and $a_{ob} > 0$ which implies an accelerated expansion for the observed space. The corresponding number of e-foldings is approximately 0.7.

It is clear that this growth is not enough for early inflation, however the metric may explain the current observational acceleration of the universe. In the solution, the internal space expands till $\tau = \tau_3 \sim 9.9$ and then it begins to contract. It is interesting to note that $\tau_3 \in (\tau_1, \tau_2)$ and thus acceleration occurs around when the internal space alters its evolution from expansion to contraction.

On the other hand, it is also possible to view the metric "backwards in time" by sending $\tau \to -\tau$. Although this does not change the acceleration, the velocity picks up a sign. In the reparametrized solution, we see that as τ runs from $-\tau_1$ to 0 we have $v_{ob} > 0$ and $a_{ob} > 0$. This is an *infinite* accelerated expansion in a finite proper time interval and the metric ends

with a singularity at $\tau = 0$. As $\tau \to 0^-$, the solution approaches to

$$ds^2 \;=\; -dt^2 \,+\, (-\tau)^{-2/3} dx^i dx^i \,+\, (-\tau)^{2/3}\, d\Sigma_p^2\,, \tag{47}$$

which is the time reversal of the vacuum Kasner metric obtained in (27) with $\epsilon = -1$ and in (35) with $\epsilon = 1$. Note that the acceleration observed in (47) is different than the power-law inflation and it would be interesting to calculate the spectrum of primordial fluctuations to compare the results with the measured spectral indices.

Accelerating cosmologies can also be obtained with brane momentum modes. After sending $t \to -t$, the metric functions for the solution (32)-(33) become

$$\begin{aligned} A &= -\frac{12}{5}\,\ln(-\sinh t) + \frac{3}{5}\,t, \\ B &= -t, \\ C &= -\frac{2}{5}\,\ln(-\sinh t) + \frac{3}{5}\,t, \end{aligned} \tag{48}$$

where in (34) the positive root[8] is picked up and we again set $b = 1$. Note that t is defined in the negative line $(-\infty, 0)$. It is possible to choose the integration constant in (21) so that as $t \to -\infty, 0$ we have $\tau \to -\infty, 0$, respectively, and thus τ is also defined in the negative line. Calculating the proper velocity and the acceleration of the observed space one finds

$$v_{ob} = \frac{dR_{ob}}{d\tau} = e^{-8t/5}\,\sinh(-t)^{12/5}, \tag{49}$$

$$a_{ob} = \frac{d^2R_{ob}}{d\tau^2} = e^{-11t/5}\,\left[\frac{12}{5}\cosh(-t) + \frac{8}{5}\sinh(-t)\right]\sinh(-t)^{19/5}, \tag{50}$$

which are always positive and thus give an accelerated expansion. Correspondingly, one can see that the internal space monotonically contracts. This evolution ends with a singularity at $\tau = 0$ where the metric becomes (47) as $\tau \to 0^-$. The expansion is limited near the asymptotic region $\tau \to -\infty$ (i.e. as $t \to -\infty$) since both v_{ob} and a_{ob} vanish. However, a large number of e-foldings can be obtained by using the solution near $\tau = 0$.

IV. CONCLUSIONS

In this work, based on the cosmological impact of brane winding and momentum modes, a stabilization mechanism for extra dimensions is proposed. We find that a gas of p-branes

[8] It turns out that the negative root does not yield acceleration.

wrapping the p-dimensional compact space can stabilize its volume as a result of a balance between winding and momentum pressures at the "self-dual" radius. This generalizes the method studied in [19] to p-branes with $p > 1$. As discussed in the introduction, a mechanism involving p-branes is desirable since strings cannot keep the volume of a generic internal manifold constant for topological reasons. On the other hand, one should also bear in mind that it is difficult to build late time realistic scenarios involving p-brane gases with $p > 2$, since the higher dimensional branes are likely to annihilate in the early universe.

Adding matter, we find that the fate of the stabilization depends on the equation of state $p = \omega\rho$ in the observed space. For $\omega > 0$, the matter can be ignored at late times and for $\omega = 0$ the self-dual radius is modified but stabilization can still be achieved. When $\omega < 0$, the late time dynamics is dominated by matter and one finds an expanding internal space, which shows that the stabilization mechanism does not work.

In this paper, we also construct accelerating solutions driven by brane winding and momentum modes. For winding modes, a short period of acceleration can be observed when the evolution of the internal space changes from expansion to contraction. This suggests an interesting mechanism for acceleration which deserves further study. Since the number of e-foldings for the expansion is small, this metric cannot be used to explain the early inflation.

In some solutions there are also accelerating periods ending with a naked singularity, which can be seen both for winding and momentum modes. Here, a large number of e-foldings can be obtained as one approaches the singularity, where the solution becomes one of the vacuum Kasner metrics. Such backgrounds may be useful in realizing inflation in brane gas cosmology, however it is necessary to find a way of resolving the singularity first, which may also explain the exit from inflation. Since the internal space shrinks to zero size, the momentum modes are expected to become important. Recall that in some solutions the metric evolves through the singularity in the presence of linearized momentum modes. However, the small-field approximation should brake down near the singularity and the full momentum modes might stop the contraction. T-duality invariance of string theory may also play a crucial role here, so it would be interesting to include the dilaton into the system. In any case, the quantum gravitational effects are expected to cure the singularity, so one can try to cut the solution and paste it to some other metric to describe the subsequent

evolution. Studying intersecting branes might also be important in this context.

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